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Research paper

FabricS: A user-friendly, complete and robust software for particle shape-fabric analysis

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ABSTRACT

Shape-fabric is a textural parameter related to the spatial arrangement of elongated particles in geological samples. Its usefulness spans a range from sedimentary petrology to igneous and metamorphic petrology. Independently of the process being studied, when a material flows, the elongated particles are oriented with the major axis in the direction of flow. In sedimentary petrology this information has been used for studies of paleo-flow direction of turbidites, the origin of quartz sediments, and locating ignimbrite vents, among others. In addition to flow direction and its polarity, the method enables flow rheology to be inferred. The use of shape-fabric has been limited due to the difficulties of automatically measuring particles and analyzing them with reliable circular statistics programs. This has dampened interest in the method for a long time. Shape-fabric measurement has increased in popularity since the 1980s thanks to the development of new image analysis techniques and circular statistics software. However, the programs currently available are unreliable, old and are incompatible with newer operating systems, or require programming skills. The goal of our work is to develop a user-friendly program, in the MATLAB environment, with a graphical user interface, that can process images and includes editing functions, and thresholds (elongation and size) for selecting a particle population and analyzing it with reliable circular statistics algorithms. Moreover, the method also has to produce rose diagrams, orientation vectors, and a complete series of statistical parameters. All these requirements are met by our new software. In this paper, we briefly explain the methodology from collection of oriented samples in the field to the minimum number of particles needed to obtain reliable fabric data. We obtained the data using specific statistical tests and taking into account the degree of iso-orientation of the samples and the required degree of reliability. The program has been verified by means of several simulations performed using appropriately designed features and by analyzing real samples.

1. Introduction

The fabric of a rock is defined by the geometric organization of the structures in the rock (Twiss and Moores, 1973). It includes the preferred orientation of the crystallographic axes of minerals, inequant clasts and crystal grains (microfabric), and the preferred orientations of fold axes, foliations, lineations and joints (macrofabric).

There are different types of fabric and different processes that produce the iso-orientation (here defined as a preferential orientation, with certain variability, towards the same direction) of the inequant elements (i.e. clasts, crystals, bubbles, joints). For example, in igneous rocks the directional fabric is due to the viscous hydrodynamic of the magma, which produces alignment of the phenocrysts, elongated xenoliths or bubbles (Herrero-Bravera et al., 2001). The fabric of a metamorphic rock thus results from the combined effects of mineral reactions and slow deformation throughout the metamorphic event (Spry, 1969). In sedimentary rocks, the fabric that develops depends on the depositional environment and the different types of interaction that can occur between particles (Mulchrone and Meere, 2015).

Fabric is a powerful tool in various disciplines of geology and enables

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inferences about the directions, magnitude and rate of displacements of forces that have acted on the rock. It has multiple applications in different fields. In this work we will focus on the applications to sedimentary petrology.

Fabric is a fundamental textural parameter for interpreting depositional processes in sedimentary deposits. It depends strongly on the physical processes occurring at the time of deposition and may also be affected by post-depositional reworking of sediments (Nichols, 1999; Boggs, 2009). Shape-fabric is a description of the three-dimensional arrangement and orientation of elongated particles. During the depositional process the elongated particles, immersed in a field of forces determined by the particle interactions, are oriented along their longest axes according to the flow direction (Capaccioni and Sarocchi, 1996; Capaccioni et al., 2001; Mulchrone and Meere, 2015). Particle orientations, analyzed by circular statistics, provide information about paleocurrent directions and polarity of movement, the influence of obstacles, and rheological behavior, among others (Potter and Pettijohn, 1977; Branney and Kokelaar, 1992; Jennings and Lucia, 2001; Zhu et al., 2004; Boggs, 2009).

Shape-fabric can be measured in several ways, from the simple use of a compass to more sophisticated methods such as anomaly magnetic susceptibility (Benn, 1994; Gautam and Rösler, 1999; Pike et al., 2001; Zhu et al., 2004), but in recent years new methods based on digital image processing have become popular (Heilbronner, 1992; Benn, 1994; Capaccioni and Sarocchi, 1996; Capaccioni et al., 1997, 2001; Gaillot et al., 1997; Lindqvist and Åkesson, 2001; Beggan and Hamilton, 2010). Using image processing, orientation can be measured by autocorrelation (Heilbronner, 1992), the intercept method (Launeau and Robin, 1996), wavelet transformation (Gaillot et al., 1997) or x-ray tomography (Jerram and Higgins, 2007). A particularly good method is the “particle by particle” method, which uses image analysis. This method is easy to use, accurate, and low-cost. The method consists of taking oriented samples (3D fabric) or photographs of the outcrop (apparent fabric), reorienting them according to the original position in the field, and analyzing them by means of circular statistics (Schäfer and Teyssen, 1987; Capaccioni and Sarocchi, 1996; Capaccioni et al., 1997, 2001; Lindqvist and Åkesson, 2001; Valentini et al., 2008; Beggan and Hamilton, 2010), which deals with directional data analysis and its representation as vectors around a unit circle.

Current fabric analysis software has some limitations. For example, some of the programs are specialized in statistical data analysis, with little or no facility for image acquisition, preparation or analysis; others are very simple and only provide rose diagrams. Other programs require specific programming skills for their use, and others are very rigorous and simpler, but obsolete and do not run on modern operating systems. For all these reasons we decided to develop a new, more complete, user-friendly software.

FabricS software was developed in the MATLAB environment. It is not open source and currently runs on the Windows operating system (Windows 7, 8 and 10) because most of the scientific community is using this system (as soon as possible we will extend the license to Mac and Linux operating systems). The FabricS program combines image analysis procedures and circular statistics algorithms. It has a graphical, easy-to-use interface and basic functions for image processing. It contains filters and thresholds for selecting particles based on their size and elongation. The reference orientation (north, horizon or other) can be adjusted precisely in each photograph. A circular statistics study of the particles is available, providing the mean orientation vector and its length (proportional to the iso-orientation strength). FabricS includes a function for loading several images from different slices of the same sample (same orientation). This makes it possible to analyze the particles all together, obtaining a single, more accurate result. FabricS processing and visualization tools are detailed in Section 2.2.

FabricS software has been tested using synthetic images with well-known features and orientation distributions. These distributions have been generated using software specifically developed for the purpose (see description in the supplementary material). In order to compare simulated and observed data we used the Kuiper two-sample test. This statistical test measures the distance between two distributions to find out how similar or different they are. Large values of the Kuiper test statistic indicate that the distributions are quite different, while small values mean that the distributions are similar. (Mardia et al., 1996). In this case, the Kuiper test shows that the two distributions, simulated and observed, are not different from each other. We also tested the software on images of three real samples in order to provide examples of using the software in both optimal and unsuitable conditions. In these cases we used the Tukey and Rayleigh tests, which are useful for characterizing circular distributions. The two statistical tests confirm the uniformity (randomness) of a circular data set based on the observed squares of deviation from the population mean. The Tukey test also considers the expected variance, and the Rayleigh test asks how large the resultant length must be to indicate a non-uniform distribution (Mardia et al., 1996). Finally, in the paper we show how the software enables several images to be analyzed, producing a single final result. This function is very useful when the single slices are small and a larger number of particles is required to achieve a certain confidence level.

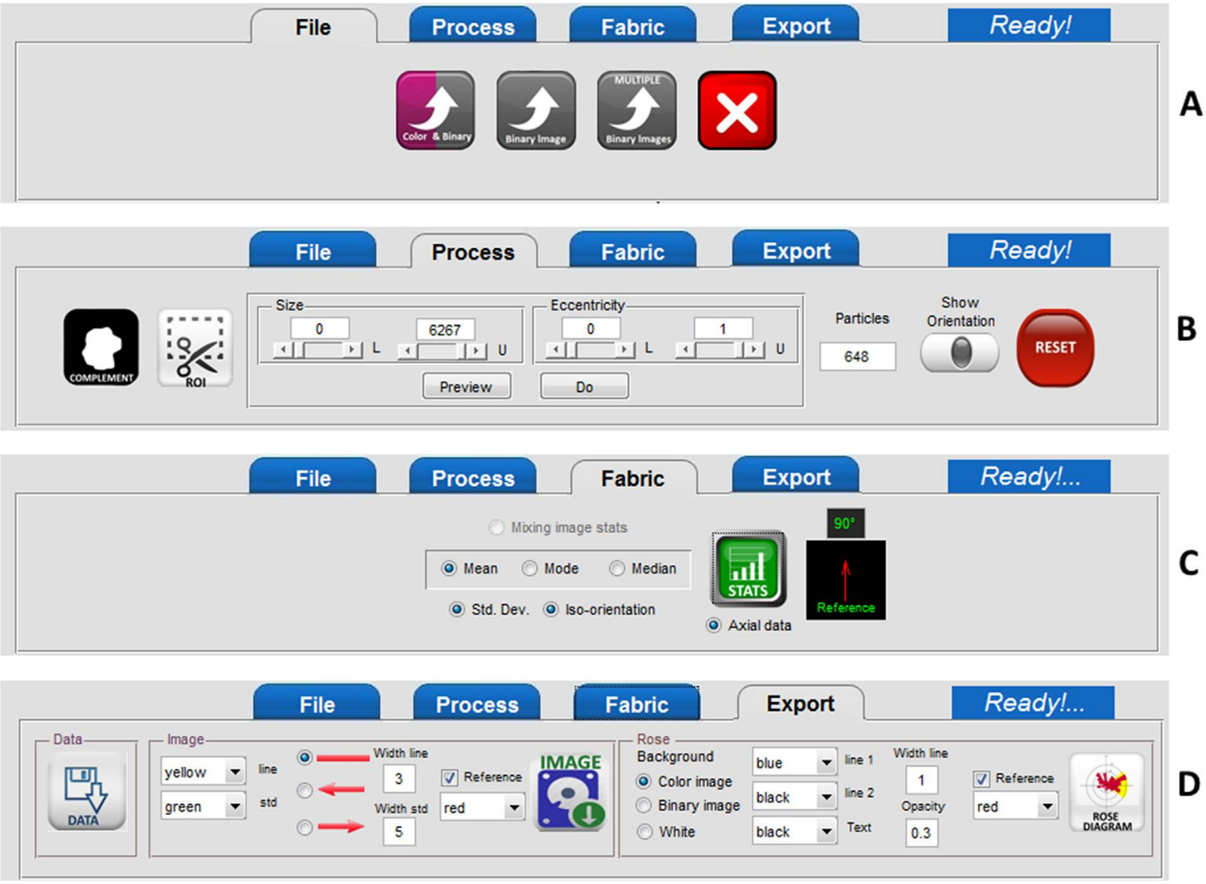
2. Materials and methods

In this section, we show different applications of fabric analysis such as apparent fabric and 3D fabric. The first is measured directly on high-resolution images of the deposit; the second requires an oriented and consolidated sample. Here we describe a sampling method useful for taking oriented compact samples from unconsolidated deposits using casting resins. Both apparent and 3D fabric are helpful for textural analysis, providing information about flow direction and polarity of the movement, and enabling inferences about the rheology. In this section, we will provide some definitions of the circular statistics useful for describing the sample. In addition, the use of software and some considerations about applications to real cases are detailed.

2.1. *Fabric data types and sampling techniques*

There are two different types of fabric in a sedimentary deposit. One is the apparent fabric, which is studied on the outcrop surface. It is not related to the true spatial orientation of clasts but to their orientation in the outcrop section. This orientation depends on the direction of the outcrop and generally is not sufficient to determine the absolute flow direction. However, it can provide information about flow polarity (origin of the flow), and indicate flow lines in vertical planes and how they are deviated by obstacles or due to turbulence. It can also be used to analyze clast arrangement in a section of a channel. The apparent fabric can be studied through high resolution photographs. In contrast, 3D fabric provides absolute information. It yields the resulting vector of the overall particle orientation in the sample. Not only does it indicate flow direction and particle imbrication but also flow polarity and iso-orientation strength (preferred orientation) and enables inference of flow rheology at the time of deposition.

Conducting a 3D fabric analysis requires an oriented sample collected in the field. The sample is cut into slices and photographs are taken of polished surfaces. Generally the slices are cut along three perpendicular directions; first, along the horizontal plane where mean flow direction can be measured. They are then cut along successive vertical planes parallel and perpendicular to the flow direction in order to spatially reconstruct the resulting vector. The samples have to be spatially oriented to provide information. The method used to collect spatially oriented samples is that proposed by Prior et al. (1987) and modified by Capaccioni and Sarocchi (1996). When the deposit is loose, the samples have to be compacted using casting resins. Then, in the laboratory, in order to facilitate resin penetration into the sample, the consolidation process is carried out in a vacuum tank. Once they have been consolidated, the samples must be photographed or scanned with the best



Size: 0 L, 6267 U
Eccentricity: 0 L, 1 U
Particles: 648

○ Mixing image stats
● Mean ○ Mode ○ Median
● Std. Dev. ● Iso-orientation
● Axial data

Image: yellow line; green std
Width line: 3
Width std: 5
☑ Reference: red
Background: ● Color image ○ Binary image ○ White
line 1: blue
line 2: black
Text: black
Width line: 1
Opacity: 0.3
☑ Reference: red

Fig. 1. Graphical interface of FabricS software. (A) Image upload tools; (B) tools related to threshold filters for particles subset selection; (C) tools for selecting circular statistics analysis and data display options; (D) controls for data and graph export options.

possible resolution, then enhanced by image processing and finally segmented. The segmented images are analyzed with the FabricS program.

2.2. Framework for directional statistics

Circular statistics is a subfield of linear statistics, and deals with the collection, analysis, interpretation, presentation, and organization of circular data (CD). CD arise in various ways and are routinely obtained in widely different scientific disciplines including astronomy (Jupp, 1995), medicine (Downs and Liebman, 1969; Gould, 1969), psychology (Gordon et al., 1989), physics (Rayleigh, 1919; Beckmann, 1959), biology (Schmidt-Koenig, 1965; Batschelet, 1981), image analysis (Mardia et al., 1996; Hanbury, 2003; Davis and Sampson, 1986) and earth sciences (Pincus, 1953; Curray, 1956; Watson, 1968; Borradaile, 2003; Carniel et al., 2017). Image analysis is used to extract circular data and directional fabric is related to the measure of particle orientation (Sestini and Pranzini, 1965; Capaccioni and Sarocchi, 1996; Capaccioni et al., 1997). In some cases, the observed directional data resulting from image processing are called axial data. This type of data could be defined as the angular position of random lines which do not have natural orientation associated with them, or in which neither end can be identified as the starting point. They are measured in terms of angles in radians (degrees), with a range of possible values (0, π] or (0°, 180°]. This is the case in the present work. To obtain an accurate interpretation, axial data can be transformed to circular data by ‘doubling the angles’ (Mardia and Jupp, 2008) and proceeding with the mathematical formalism as follows (see Berens, 2009).

Let o be a particle and $O = \{\mathrm{o_j} | \, \mathrm{e(o_j)} \geq \mathrm{l}, \mathrm{j} = 1, 2, 3, \ldots, \mathrm{N}\}$ be the set of N particles within the image, where $e(o)$ is the elongation of each particle which is estimated by approximating the particle to an ellipse and calculating the ratio between the distance from the center to the focus and the distance from the focus to the vertex. The l parameter is important because spherical or circular data lack orientation and therefore fabric information is not present. Now, θ is the azimuth angle of particle o with respect to a predefined reference vector (north) and the set of directional data $A = \{\theta_j, j = 1, 2, 3, \ldots, N\}$. Also, as noted above, another consideration is that the set of θ are axial data. This is for two reasons; there is no front or polarity in a rock, and it is assumed that the rock follows the direction of the flow. Due to the former, the rose diagram is often plotted symmetrically. When working with axial data, angles are multiplied by 2 in order to calculate certain statistical parameters (Mardia and Jupp, 2008). Although this procedure is not the best way to estimate such parameters (Arnold and SenGupta, 2006), it is accepted within the field of circular statistics.

Now, θ is represented as a vector of circular data in a circular data plot, where each θ_j is plotted as a single point on the unit circle.

Each of the points “x” can be represented as a function of θ. These are related to x by

$$x = \left(cos\theta_j, \sin\theta_j\right). \tag{1}$$

The mean length of the resulting vector is given by $\overline{R} = \sqrt{\overline{C}^2 + \overline{S}^2}$ where $(\overline{C}, \overline{S})$ are the Cartesian coordinates of the points “x”, which can be estimated by

$$\overline{C} = \sum_{j=1}^{N} cos\theta_j \quad \overline{S} = \sum_{j=1}^{N} sin\theta_j. \tag{2}$$

The mean resultant vector length $\overline{R}$ takes values in the range $[0, 1]$. If

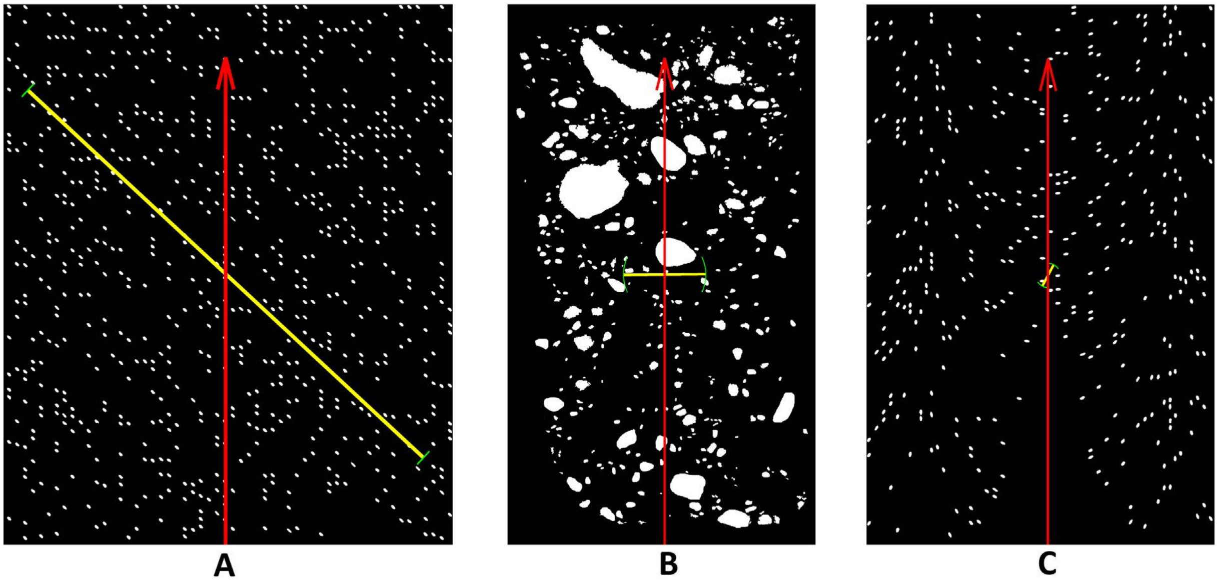


Fig. 2. Figures with direction reference (red arrow), iso-orientation vectors (yellow line) and standard deviation arcs (green arcs). Figure A represents a sample with a mean direction of 133° and a vector length of 0.99 (very good iso-orientation). Figure B represents a sample with a mean direction of 90° and a vector length of 0.24 (medium to low degree of iso-orientation). Figure C shows an average direction of 28° and a vector length of 0.05 (very low degree of iso-orientation). (For interpretation of the references to color in this figure legend, the reader is referred to the Web version of this article.)

the directions θ are tightly clustered, then $\overline{R}$ will be almost 1 (Mardia and Jupp, 2008), and when these directions are spread around the circle, $\overline{R}$ is closer to 0, indicating a uniform distribution around the circle. Thus the value of $\overline{R}$ is indicative of the concentration of a data set.

We can also define mean direction as the direction of $\overline{R}$,

$$\overline{\theta} = \begin{cases} tan^{-1}\dfrac{S}{C} & C \geq 0 \\ tan^{-1}\dfrac{S}{C} + \pi & C < 0 \end{cases} \tag{3}$$

Note that this definition is completely different from the conventional mean direction ($\overline{\theta} = \frac{1}{N}\sum_{j=1}^{N}\theta_j$), where mean direction is the sum of all directions divided by the total number of directions. Details and other statistical parameter definitions can be found in (Fisher, 1995; Mardia and Jupp, 2008).

In order to make the right decision about the concentration of the data we use Tukey and Rayleigh statistical tests to make inferences about the data; namely, whether the data have a predominant orientation or not. This inference will be made with a significance level of 5% and is compared to the notion of concentration given by $\overline{R}$.

2.3. FabricS software description

The FabricS program consists of four sections. These sections correspond to the divisions in the tabbed interface. Fig. 1 shows each of the tabs, with the tools in the corresponding window.

In the first section (Fig. 1A, File), binary and color images can be loaded. Measurements are made on binary images, previously segmented with dedicated software. In order to segment color images, we suggest using OPTGRAN-CS (Moreno-Chávez et al., 2015). Color images can be uploaded as reference images in order to compare and check the features present in the segmented images, but they are not required because measurements are performed on the binary images.

FabricS allows multiple images of the same sample to be uploaded and can provide unified statistics and diagrams considering the analyzed particles to belong to the same area. This is particularly useful for analyzing sliced samples.

The second section (Fig. 1B, Process tab) corresponds to a preprocessing block, where the region of interest (ROI) can be chosen. The color (black or white) of features to be analyzed can be selected. The user can define the filter thresholds (size and eccentricity). The "Particles" window shows the total number of particles included in the selection. The "show orientation" button superposes orientation data on the binary image as small line segments. This tool is useful for visualizing iso-orientation.

The use of these thresholds is crucial because they define the set of particles that will be measured. While the elongated particles are oriented by a field of forces acting within the flow during movement, spherical particles, or those with approximately isotropic shape, are not oriented or are less sensitive to iso-orientation. The use of a low eccentricity threshold can therefore result in a biased analysis. In practice we observed that $0.5 < e(o) < 0.9$ is usually a reasonable eccentricity threshold range.

Size threshold is also very important because particles of different sizes can be sensitive to different fields of forces. Small particles tend to be affected by local turbulence or deflected by bigger clasts. In contrast, larger particles are less sensitive to local fluctuation and can be more indicative of flow path. However, coarse particles are less abundant than fine ones and there might not be enough for a proper statistical analysis. The smallest particles selected must have an area of at least 12 pixels because in these particles the smallest areas are at the noise limit, as could be observed experimentally in our tuning tests and as it has been mathematically proved in the papers of Cuevas et al. (2010) and Kröner and Doménech Carbó (2013).

The third section (Fig. 1C, Fabric) involves statistical calculations and visual presentation. As explained above, clast orientation angles are presented as axial data, but the program includes an option for dealing with either axial or circular data. The program calculates the mean, median and mode direction, standard deviation, resultant vector length, number of oriented particles measured, and Rayleigh and Tukey statistical test results. Useful visual tools are the resultant vector length and standard deviation. These tools enable an easy and accurate interpretation of the iso-orientation of the particles. The vector length is a line (vector) whose length is directly proportional to the concentration data (degree of iso-orientation of clasts) and its orientation. Mean, median or mode can be chosen. The standard deviation is represented by two circle sectors with angular amplitude corresponding to the standard deviation.

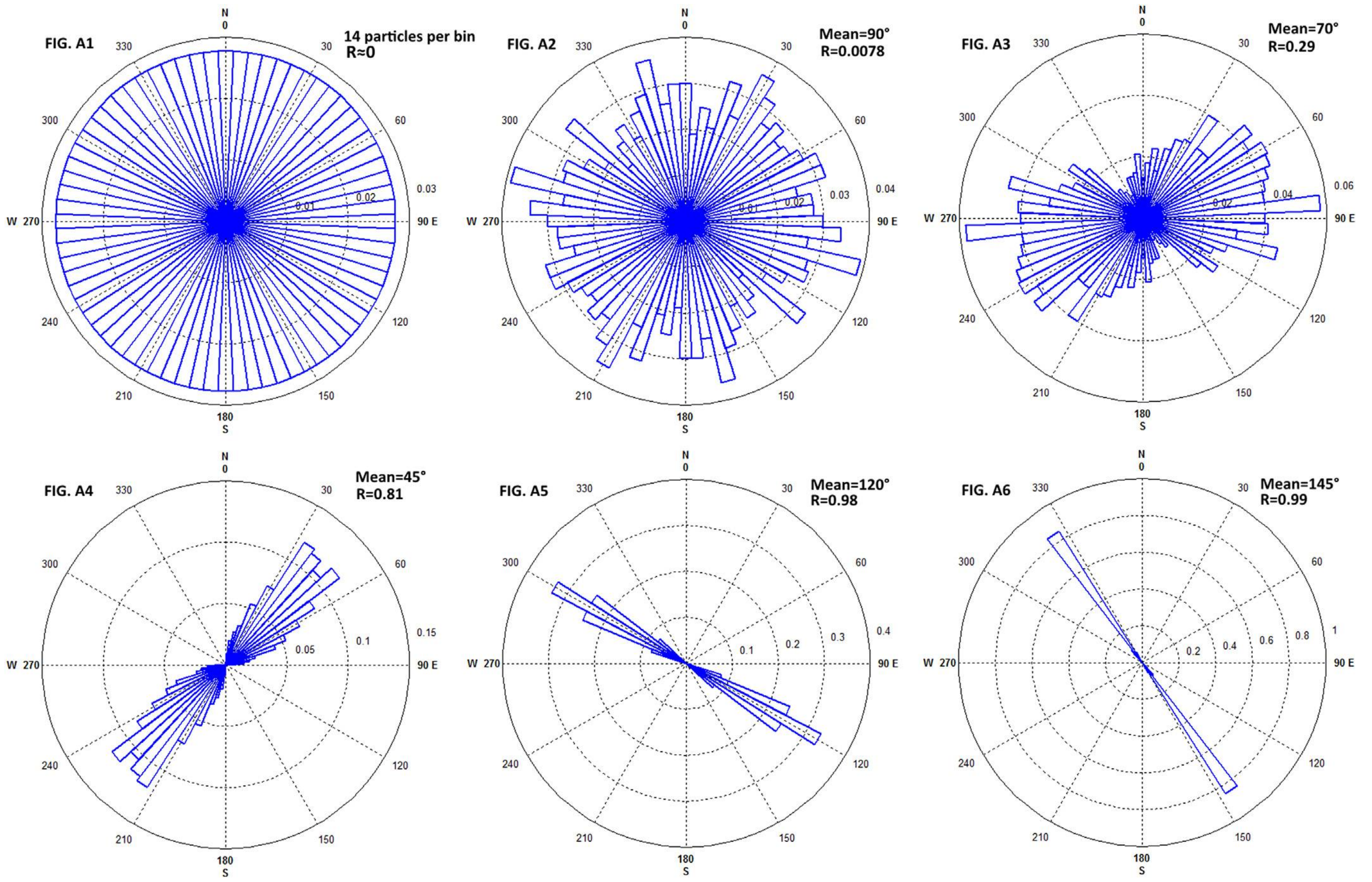


Fig. 3. Rose diagrams obtained by FabricS, analyzing six synthetic distributions characterized by different degrees of iso-orientation (different concentration parameter R). All are based on the analysis of about one thousand particles and range from perfectly isotropic (R = 0) to quasi unimodal (R = 0.99). R varies exponentially between 0 and 1. From values around 0.10 the anisotropicity of the distribution is clearly visible to the naked eye in a rose diagram.

In this section the direction of the reference (i.e. north or horizon) can be selected. Generally the reference is chosen with the north parallel to the left side of the image but the user can change the reference according to specific needs. Fig. 2 shows some examples.

The last block (Fig. 1D, Export) focuses on export functions. The tabular data information, generated in the third block, is exported as a file, which can be read by commercial data analysis software (Excel, Origin etc.). FabricS can export rose diagrams whose color and size can be chosen by the user. Images with color or binary features can be exported with iso-orientation vectors and standard deviation arcs. Fig. 2 shows an example. The data provided are simple directions without polarity; however, for purposes of visual representation, an arrow indicating the polarity can be set using the buttons.

In order to offer more details about the use of FabricS software we developed a user manual, freely downloadable from our website http://www.laima-uaslp.org/.

3. Results and discussion

In this section we present the results of the tests of the FabricS software using synthetic and real images, made in order to evaluate the accuracy and reproducibility of the method. We developed a special algorithm to generate synthetic images (a brief description is given in the supplementary material). Real sample images were also used in order to show the practical application of the software.

We tested how fabric parameters vary depending on the size of the particles. We also provide a guide to help the user to choose an appropriate sample size (which depends on the iso-orientation degree) to obtain accurate results.

3.1. *Test using synthetic images*

Synthetic binary images were created based on simulated distributions with known orientation and eccentricity. In order to generate the random orientation, a von Mises distribution was used. This is a continuous probability distribution on the circle and is a circular analogue of the normal distribution. It is the most common distribution used to make inferences on the circle and is also the most developed due to its elegant structure as an exponential transformation model under rotations (Mardia and Jupp, 2008). The concentration parameters (degree of iso-orientation) were adjusted so that the distributions could range from uniform to strongly oriented. The mean directions were chosen without any specific criterion because they do not affect other statistical parameters.

Fig. 3 shows the rose diagrams of six artificial distributions with different degrees of iso-orientation, generated by testing the software.

Using the OPM (Oriented Particles Maker) software (see supplementary material section) we generated artificial samples with one thousand particles whose eccentricity follows a Gaussian distribution with a mean of 0.8 and a standard deviation of 0.02. With this configuration, we generated elongated elliptical particles with known orientation. The ellipses were deformed using random morphological operators in order to be as similar as possible to real particles. Fig. 4 shows examples of artificial particles with different degrees of deformation. The figure shows that ellipses with a low degree of elongation lose their orientation faster by increasing the degree of irregularity. The complete set of images used for the test is available in the supplementary material section (Fig. S1 to S5).

Tests were performed using the six concentration degrees of Fig. 3

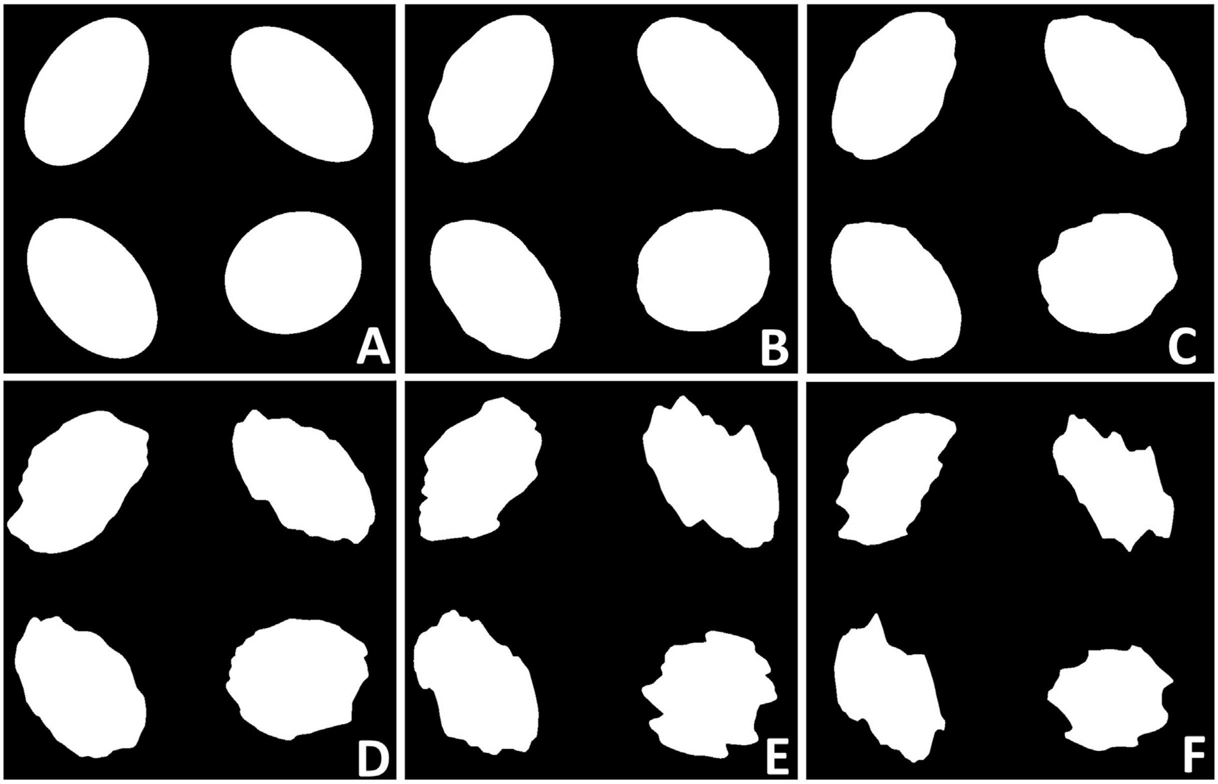


Fig. 4. Synthetic particles with different degrees of deformation, produced with the OPM (oriented particles maker) program. In spite of deformation, the orientation of the elongated particles is preserved.

Table 1
Directional statistics parameters simulated by OPM and measured by FabricS software. (* see supplementary meterial).

Image	Mean Simulated/ Observed	Resultant vector length Simulated/ Observed	Standard deviation Simulated/ Observed	p-value Kuiper test
Fig. S1*	114°/105°	1e-5/1e-5	29°/29°	0.8265
Fig. S2*	90°/65°	3e-2/3e-2	28°/28°	1
Fig. S3*	70°/70°	0.3/0.3	24°/24°	1
Fig. S4*	45°/45°	0.8/0.8	12°/12°	1
Fig. S5*	120°/120°	0.98/0.98	4°/4°	1
Fig. S6*	145°/145°	0.99/0.99	1°/1°	1

with particle irregularities corresponding to Taylor and Pettijohn's sub-rounded class (Muller, 1967).

Synthetic distributions (simulated values) and FabricS measures (observed values) were compared using a Kuiper two-sample test. For all cases, the test rejected the null hypothesis at the 5% significance level (see Table 1). These results prove that there is no evidence that the samples come from different distributions. The vector length depends on the particles' degree of iso-orientation. In Table 1, all parameters are practically the same except the mean direction of Figures S1 and S2. This discrepancy is due to the high degree of dispersion of the distributions used (see Fig. 3). The mean value of the simulated and observed data tends to be the same as the concentration of the data increases. This test shows that the software is working properly.

3.2. Evaluation of the minimum sample size

In this section, the term minimum sample size refers to the smallest set of particles of a slice (a polished section of the oriented specimen) or set of slices that provide reliable orientation data. Sample size has a significant impact on the accuracy of the calculated fabric parameters. The minimum sample size for obtaining accurate data depends on several factors and not on a single general criterion for different applications (Capaccioni and Sarocchi, 1996; MacCallum et al., 1999). The optimum sample size is the number of particles sufficient to satisfy a specified degree of certainty.

In order to determine the minimum sample size, we perform a statistical test using a unimodal distribution with different $\overline{R}$ parameter concentrations. Sample size depends on the characteristics of the statistical population, and more specifically on the iso-orientation degree. The greater the degree of iso-orientation ($\overline{R}\to 1$), the smaller the number of particles needed to obtain reliable data.

Fig. 5A shows the estimation of the mean direction (equation (3)) with respect to the sample size. For a low concentration parameter, the mean direction is random. However, mean direction converges as the concentration parameter increases. If the concentration parameter is higher it converges faster. The concentration parameter and the standard deviation have the same behavior (Fig. 5B and C). The greater the degree of iso-orientation of the particle set, the faster the parameters tend towards the final result and the smaller the number of particles that needs to be analyzed to obtain reliable results.

If it is assumed that the population has $\overline{R}\to 1$ (quasi-unidirectional distribution), then the sample size can be small. On the other hand, if the population tends to be uniformly distributed, namely $\overline{R}\to 0$, then the minimum sample size tends to be larger.

One way to establish the minimum sample size is to consider the variability of the statistical parameters. Some authors report that in order to obtain a variability of ±10° or lower, the sample size should be at least 300 particles (Capaccioni et al., 1997). However, as discussed above, the convergence (variability) of fabric parameters depends on their iso-orientation degree.

The statistical tests based on sample size and the concentration parameter R make it possible to establish quantitative criteria for

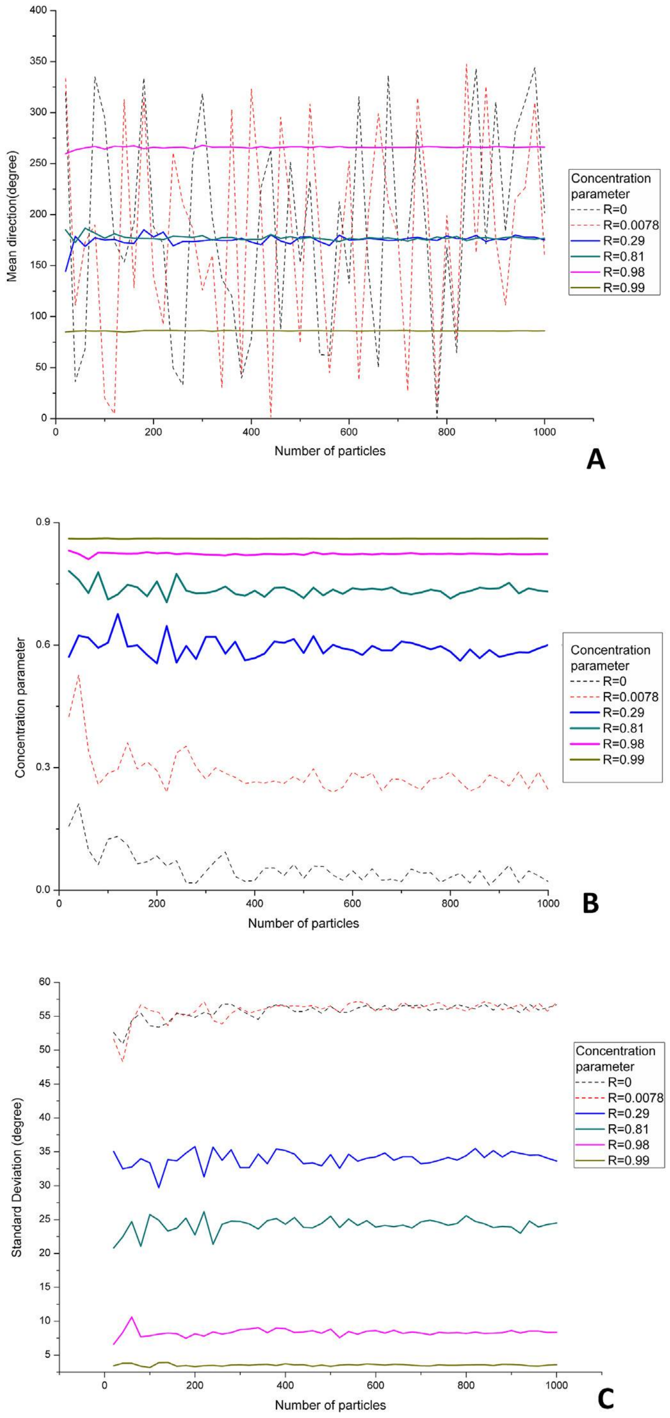


Fig. 5. This figure shows the variation in mean direction, concentration parameter (R) and standard deviation for increasing numbers of particles analyzed from six different samples. (A) Variation of mean vector direction, (B) variation of the resultant vector length (iso-orientation degree), and (C) variation of the standard deviation. The greater the iso-orientation degree of the particle set, the faster the parameters tend to the final result and the smaller the number of particles to be analyzed to obtain reliable results.

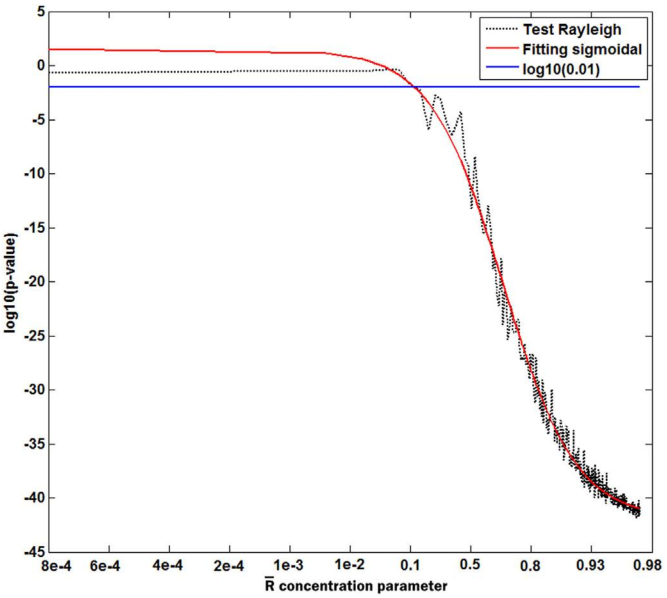


Fig. 6. Example of a log-log diagram relating the concentration parameter (R), the minimum particle number to satisfy the Rayleigh test with a confidence level set at $\alpha = 0.01$ and sample size of 100 particles.

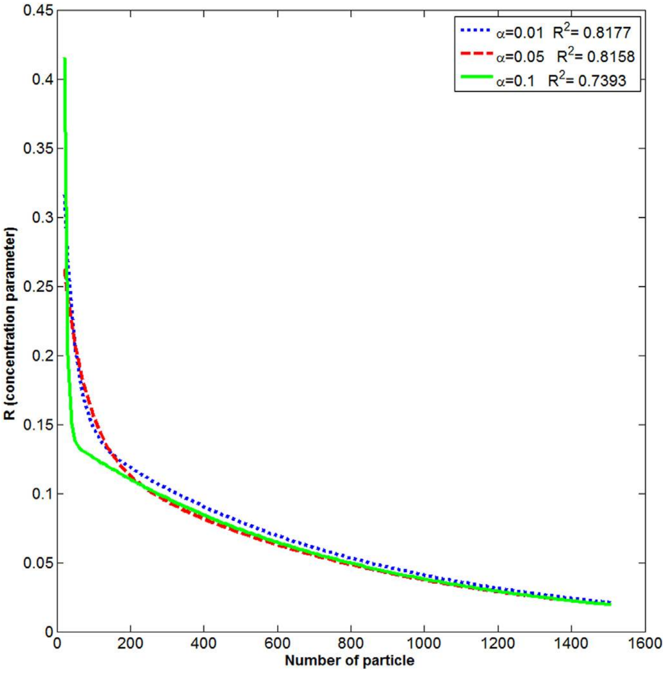
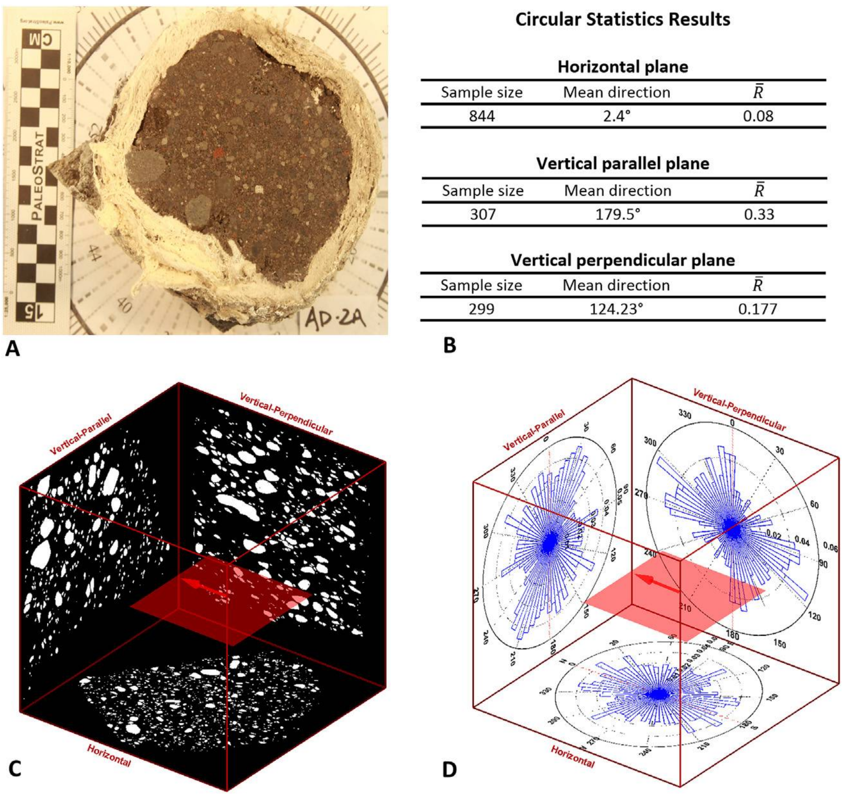


Fig. 7. Minimum concentration, given a specific sample size, at which the Rayleigh test rejects the null hypothesis for three different confidence levels ($\alpha = 0.1$ is the largest confidence level). Curves are similar with sets of particles larger than 200 units. Differences arise with a larger confidence level ($\alpha = 0.1$) and $R > 0.1$.

accepting or rejecting an analysis. To find the minimum sample size given a particular iso-orientation, the distribution must meet a standard of statistical confidence on the Rayleigh test. Random populations were generated using the von Mises distribution where the concentration parameter K and particle number were varied. To avoid confusion, we replaced the K parameter with the parameter $\overline{R}$ (directional statistics) using the expression developed by Mardia and Jupp (2008). Distributions were generated with sample sizes ranging from 20 to 1500 particles in increments of 10, and the $\overline{R}$ concentration parameter ranged from 8×10^{-4} to 0.99 in increments of 0.01. For each combination, we applied the Rayleigh test to estimate the minimum $\overline{R}$ value, given a certain sample size, at which the test rejects the null hypothesis. To calculate the minimum $\overline{R}$ value, we built the semilog diagram, plotting p-value against the $\overline{R}$ concentration parameter. The curves were fitted by means of a



Circular Statistics Results

Horizontal plane

Sample size	Mean direction	$\bar{R}$
844	2.4°	0.08

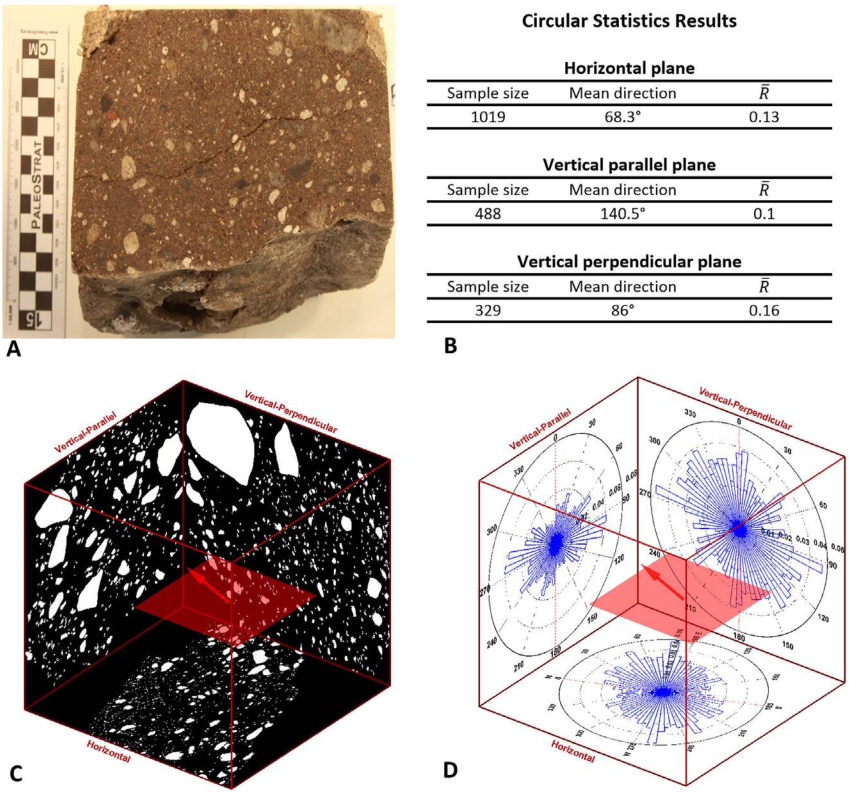
Vertical parallel plane

Sample size	Mean direction	$\bar{R}$
307	179.5°	0.33

Vertical perpendicular plane

Sample size	Mean direction	$\bar{R}$
299	124.23°	0.177

Fig. 8. 3D Fabric analysis of an oriented sample collected at Mt. St. Helens. A) Polished section of sample photographed along the horizontal plane; B) Essential statistical data related to the three different planes; C) 3D representation of the resultant vector (calculated by 3DVCV software) using an oriented cube with the segmented images on the sides. D) Representation of the resultant vector using an oriented cube and rose diagrams on the sides. This sample is characterized by a very low degree of iso-orientation along the horizontal plane (R = 0.08) with flow direction azimuth 2.4°, good degree of iso-orientation along the vertical-parallel plane to the flow (R = 0.33) with particle inclination of 179.5°. In the vertical-perpendicular plane particle inclination is 124.2° and a moderate to good degree of iso-orientation is R = 0.177.



Circular Statistics Results

Horizontal plane

Sample size	Mean direction	$\bar{R}$
1019	68.3°	0.13

Vertical parallel plane

Sample size	Mean direction	$\bar{R}$
488	140.5°	0.1

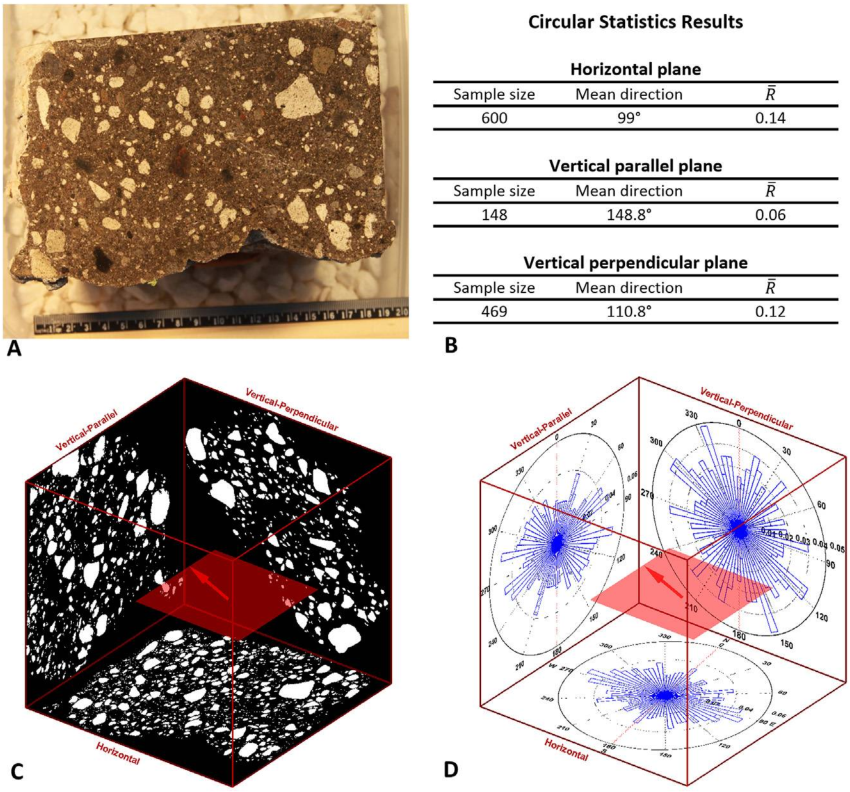
Vertical perpendicular plane

Sample size	Mean direction	$\bar{R}$
329	86°	0.16

Fig. 9. 3D Fabric analysis of an oriented sample collected at Mt. St. Helens. A) Polished section of sample photographed along the horizontal plane; B) Essential statistical data related to the three different planes; C) 3D representation of the resultant vector (calculated by 3DVCV software) using an oriented cube with the segmented images on the sides. D) Representation of the resultant vector using an oriented cube and rose diagrams on the sides. This sample is characterized by a moderate degree of iso-orientation along the horizontal plane (R = 0.13) with flow direction azimuth 68.3°, low to moderate degree of iso-orientation along the vertical parallel plane (R = 0.10) with particle inclination of 140.5°. In the vertical-perpendicular plane particle inclination is 86° and the degree of iso-orientation is moderate to good (R = 0.16).

sigmoidal function. Fig. 6 shows an example with a confidence level set at $\alpha = 0.01$.

The procedure used for the example shown in Fig. 6 was applied for three confidence levels ($\alpha = 0.1$, 0.05, 0.01) and 400 different sample sizes. Results are shown in Fig. 7. It can be seen that if the distribution has a particular degree of concentration, even if it is very small, with a sample



Circular Statistics Results

Horizontal plane

Sample size	Mean direction	$\bar{R}$
600	99°	0.14

Vertical parallel plane

Sample size	Mean direction	$\bar{R}$
148	148.8°	0.06

Vertical perpendicular plane

Sample size	Mean direction	$\bar{R}$
469	110.8°	0.12

Fig. 10. 3D Fabric analysis of an oriented sample collected at Mt. St. Helens. A) Polished section of sample photographed along the horizontal plane; B) Essential statistical data related to the three different planes; C) 3D representation of the resultant vector (calculated by 3DVCV software) using an oriented cube with the segmented images on the sides. D) Representation of the resultant vector using an oriented cube and rose diagrams on the sides. This sample is characterized by a moderate degree of iso-orientation along the horizontal plane (R = 0.14) with flow direction azimuth 99°, very low degree of iso-orientation along the vertical-parallel plane to the flow (R = 0.06) with particle inclination of 148.8°. In the vertical-perpendicular plane particle inclination is 110.8° and the degree of iso-orientation is moderate (R = 0.12).

size of 1400 particles or more, the preferential direction can be found with a confidence level of $\alpha \leq 0.01$. However, for sample sizes smaller than 1400 particles, a minimum iso-orientation degree ($\bar{R}$ parameter) is required to find a preferential orientation. Based on these facts, the optimal analysis should use more than 1400 particles. A good analysis is possible with fewer particles but the confidence level should be taken into account. In practice, if the sample size is not large enough to meet the test requirements, the oriented specimen can be cut into a larger number of slices in order to increase the total number of particles.

3.3. Tests with real 3D samples

For a better understanding of how FabricS works, in this section we show the results of the analysis of three 3D samples collected in the primary pyroclastic density current deposit from the May 18, 1980 eruption of Mt. St. Helens (Lipman and Mullineaux, 1981).

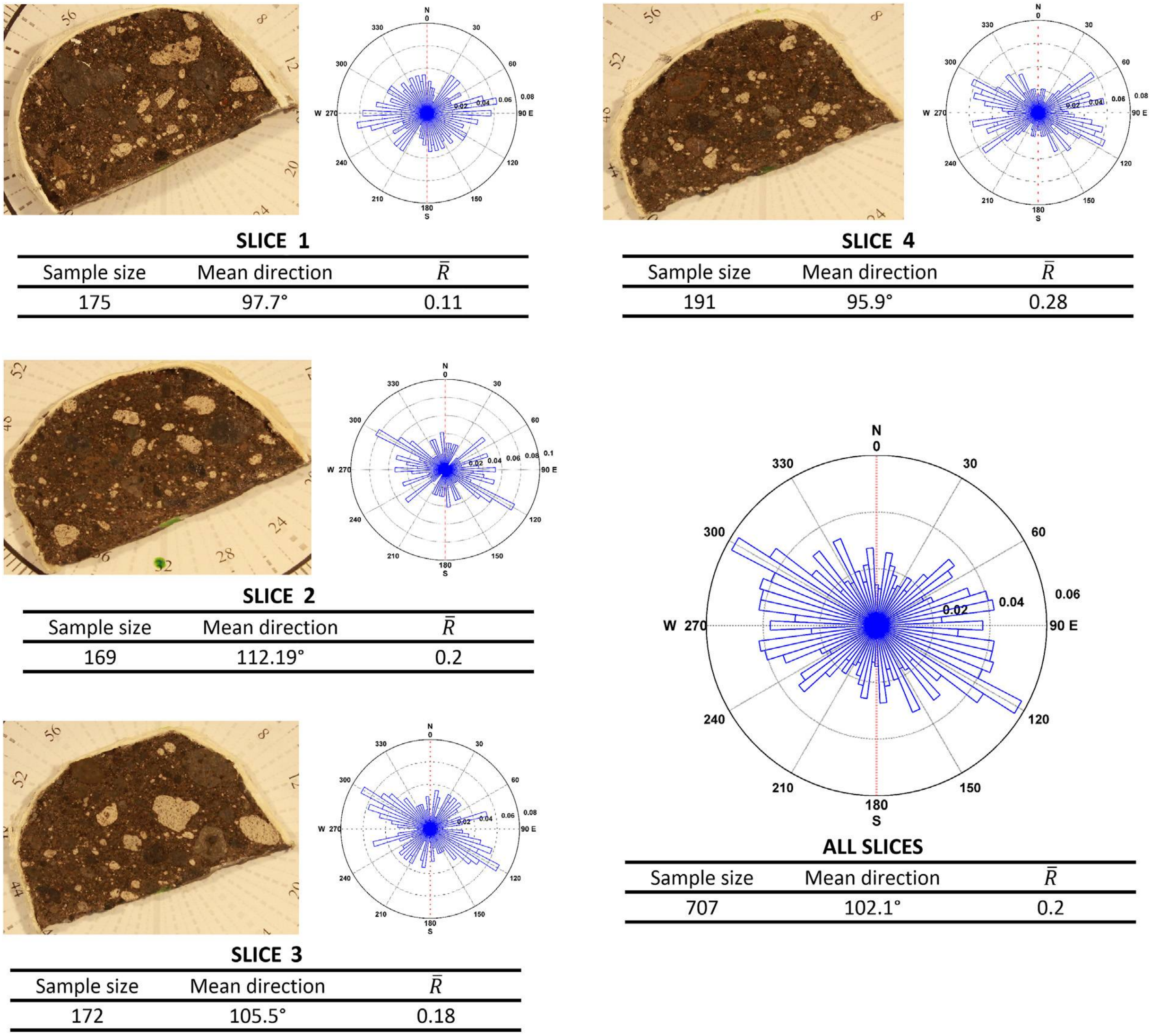
The samples were consolidated using casting resins and collected following the technique proposed by Capaccioni and Sarocchi (1996). Once the samples were consolidated, they were cut into 20-mm slices with an area of approximately 10–15 cm^2. The samples were cut along three main surfaces: 1) along the horizontal plane, 2) vertically along the flow direction (mean direction of iso-orientation in the horizontal plane) and 3) vertically perpendicular to the flow direction. The surfaces of the slices were polished, and photographed with a 15 Mpx resolution camera, taking care not to introduce any deformation. These images are shown in Figs. 8A, 9A and 10A. The color images were segmented using the freeware software OPTGRAN-CS (Moreno Chavez et al., 2015) and edited manually in order to minimize errors due to segmentation. Using the segmented images with FabricS software, we obtained the circular statistics analysis of the samples. The elongation thresholds of the particles were set at 0.5–0.6 and the area thresholds were set at >20 pixels. Figs. 8B, 9B and 10B show the results of the analysis. Parameters of sample size, mean direction in three spatial directions and the concentration parameter are reported.

Figs. 8C, 9C and 10C show a cube that represents the three spatial planes of the analysis with the corresponding binary image superimposed. The vectors resulting from the analysis in the three spatial directions are inscribed in the cube. The direction of the horizontal and/or the reference plane are also shown. The vector length depends on the degree of iso-orientation of the particles in each plane. The cube is constructed using 3DVCV software (3D Vector Calculator and Visualizer), dedicated software developed in our laboratories and described in the reference material section. The program is freeware and available at our website (http://www.laima-uaslp.org/descargas.html). Similarly, Figs. 8D, 9D and 10D show the cube with the rose diagrams superimposed.

In order to have statistical reliability, a sufficient sample size and concentration must be used as shown in Fig. 7. The three planes of Fig. 8 are statistically significant at $\alpha = 0.01$. Therefore the real sample 1 analysis has a high level of confidence. For the results shown in Fig. 9B, the three planes are statistically significant at $\alpha = 0.01$. Hence, the real sample 2 analysis can be accepted with confidence. For the real sample 3 shown in Fig. 10, the horizontal and vertical perpendicular planes have a confidence level of $\alpha = 0.01$ but the vertical parallel plane has $\alpha < 0.1$. This last analysis should be treated as unreliable because of the low confidence in the parallel plane.

Considering the three examples shown, sample 3 is not statistically reliable. In contrast, samples 1 and 2 show no evidence of randomness. The file exported from FabricS software gives the *P*-value of the Tukey and Rayleigh statistical tests. For the three planes of the real samples 1 and 2, the Rayleigh and Tukey test have $P < 0.01$, which means that there is no evidence of randomness in the distributions. The results of the FabricS software have been validated by the uniformity test and the sample size graph of Fig. 7.

When a single slice of the sample is not enough to provide the minimum number of particles, the program allows multiple surfaces from the same sample to be uploaded, and analyzes them all together providing a single statistic and exporting the results in a single graph. This option is useful in those cases where the sample size from a single slice is not statistically reliable (small number of particles). Fig. 11 shows an



SLICE 1

Sample size	Mean direction	$\bar{R}$
175	97.7°	0.11

SLICE 2

Sample size	Mean direction	$\bar{R}$
169	112.19°	0.2

SLICE 3

Sample size	Mean direction	$\bar{R}$
172	105.5°	0.18

SLICE 4

Sample size	Mean direction	$\bar{R}$
191	95.9°	0.28

ALL SLICES

Sample size	Mean direction	$\bar{R}$
707	102.1°	0.2

Fig. 11. Analysis of different slices, parallel to each other, cut sequentially from the same sample. Slice #1 does not satisfy the Rayleigh test and it has to be discarded. Slices #2 to #4 satisfy the test and show a good degree of iso-orientation of the particles (R = 0.2). The analysis of the whole set of slices (707 particles) provides reliable data very close to the orientation provided by slices #2 to #4.

example of a multiple slice analysis. Slice #1 is not statistically significant, due to the small sample size. This can be resolved by cutting more slices parallel to the first one so as to increase the analyzed surface and consequently the number of particles in the sample. The analysis of the other slices (#2, #3 and #4) are significant at $\alpha = 0.01$, as well as the analysis of all slices of the sample, which are also significant at $\alpha = 0.01$. Following this method can increase statistical confidence without requiring an additional sample.

Once the samples meet these validation criteria, they can be used for sedimentological studies and geological interpretation.

4. Conclusion

FabricS is a user-friendly software package with a graphical interface used to obtain shape-fabric information by means of rigorous circular statistics algorithms. Although the study of shape-fabric in sedimentary deposits has been the main objective of this work, the software can be very useful for studying the fabric in different fields of petrography (i.e., orientation of elongated crystals in paragenesis or deformed and recrystallized objects in metamorphic rocks) and in all fields of science and technology where it is necessary to quantitatively study the direction of elongated objects.

The software provides results in the form of data tables or graphics, both easy to interpret. The algorithm calculates the central tendency values of the particle set (mean, median, and mode), the standard deviation, and the degree of iso-orientation. All these data can be visualized in the image through a simple and easy-to-interpret symbology that simplifies the analysis and interpretation of circular data.

An important contribution of this work is the calculation of the minimum number of particles needed to obtain reliable data, as a function of the degree of iso-orientation. This result was obtained by means of a simulation process with artificial features and a rigorous statistical study based on the Rayleigh statistical test. These simulations clearly indicate that the speed of convergence of the data has a dependent relationship with the central tendency of the orientation parameter and the specific degree of iso-orientation of the sample. The higher the degree of iso-orientation of the samples, the lower the number of measurements needed to obtain a reliable iso-orientation direction. The study indicates that with 850 or more measurements, the iso-orientation direction, if it exists, can reliably be found even if it is weak. It was also observed that in most cases a sample of 300–400 particles is sufficient to provide reliable data, as previously reported in the literature.

The FabricS software package was developed in the MATLAB environment and runs on Windows 7 or higher. The proposed method and the software are original and unique and will enable shape-fabric to be studied more easily and reliably.

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Appendix A. Supplementary data

Supplementary data related to this article can be found at https://doi.org/10.1016/j.cageo.2018.02.005.

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